Determination of Young's Modulus of a metallic bar using traveler pulse detection by fiber optic sensors type LPG

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Abstract– Two fiber optic accelerometers LPG were glued on the surface of a metal rod simply supported at its ends. Transverse and longitudinal pulses were excited at one end of the rule. The sensors were placed near the ends of the rod. Step pulse is detected by each LPG sensor. Measuring the distance between the sensors and travel time pulse, the transmission speeds longitudinal and transverse pulses are determined, from which is the estimated value of the elastic modulus of the metal rod.

I. INTRODUCTION

Knowing the characteristics of elasticity of materials is a task of utmost importance in civil engineering. The Young's modulus is one of the important parameters of the material since it determines its load resistance [1]. Young's modulus can be obtained from static or dynamic measurements. Although the Young's modulus can be measured dynamically using electrical sensors [2,3], fiber optic sensors have different advantages such as their small size and immunity to electromagnetic interference [4,5]. This article shows the use of optical fiber LPG sensors to determine the Young's modulus of an aluminum metallic bar, measuring the propagation velocities of longitudinal and transverse pulses along the bar via the optical response of the LPG sensors.

A. Theory

The general relationship between the applied stresses and the deformation of an elastic material is given by the following expression [6]:

$$t_{ij} = C_{ijkm} e_{km} \tag{1}$$

Which is known as the generalized Hooke's law. t_{ij} is the stress tensor, e_{km} is the strain tensor, and C_{ijkm} is the elasticity tensor.

$$t_{11} = t_1;\ t_{22} = t_2;\ t_{33} = t_3 \tag{2}$$

$$t_{23} = t_{32} = t_4\ ;\ t_{13} = t_{31} = t_5;\ t_{12} = t_{21} = t_6 \tag{3}$$

As well,

$$e_{11} = e_1;\ e_{22} = e_2;\ e_{33} = e_3 \tag{4}$$

$$e_{23} = e_{32} = e_4\ ;\ e_{13} = e_{31} = e_5;\ e_{12} = e_{21} = e_6 \tag{5}$$

From where:

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} \tag{6}$$

In isotropic materials:

$$t_{ij} = l d_{ij} e_{kk} + 2u e_{ij} \tag{7}$$

$$e_{ij} = \frac{1}{2u}\left(t_{ij} + \frac{l}{3l+2u} d_{ij} t_{kk}\right) \tag{8}$$

In matrix form:

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \end{bmatrix} = \begin{bmatrix} l+2u & l & l & 0 & 0 & 0 \\ l & l+2u & l & 0 & 0 & 0 \\ l & l & l+2u & 0 & 0 & 0 \\ 0 & 0 & 0 & u & 0 & 0 \\ 0 & 0 & 0 & 0 & u & 0 \\ 0 & 0 & 0 & 0 & 0 & u \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix} \tag{9}$$

Where, l and u are the constants of Lamé. The Lamé constants are related to the Young's modulus E and the Poisson's ratio s through the following relationships:

$$E = \frac{u(3l+2u)}{l+u} \tag{10}$$

$$s = \frac{l}{2(l+u)} \tag{11}$$

The relations of the velocities of the sound inside the material with the constants of Lamé, are given by:

$$l = d(c_l^2 - 2c_t^2) \tag{12}$$

$$u = dc_t^2 \tag{13}$$

So, the elastic constants C_{11} and C_{44} are related to the transverse velocity c_t and longitudinal c_l of the sound propagating in the isotropic material in the following way:

$$C_{11} = dc_l^2 \tag{14}$$

$$C_{44} = dc_t^2 \tag{15}$$

Where, d is the density of the material.

An additional constant of importance in engineering is the rigidity modulus (G), defined as:

$$G = \frac{E}{2(1+s)} = u \tag{16}$$

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From the previous relationships, Young's modulus (E), rigidity modulus (G) and Poisson's ratio (s) are expressed as a function of longitudinal (c_l) and transverse (c_t) acoustic wave velocities:

$$E = \frac{3dc_t^2\left(c_l^2-\frac{4}{3}c_t^2\right)}{\left(c_l^2-c_t^2\right)} \quad (17)$$

$$G = dc_t^2 \quad (18)$$

$$s = \frac{\left(c_l^2-2c_t^2\right)}{2\left(c_l^2-c_t^2\right)} \quad (19)$$

II. EXPERIMENT

Figure 1 shows the experimental arrangement used to measure the velocities of the transverse and longitudinal pulses propagating in an aluminum bar simply supported at its ends. Two LPG grating written in SMF28 commercial mode mono optical fiber, using CO_2 laser, are located near the ends of the 3 m long bar. Transverse and longitudinal pulses are originated in the bar hitting one of its ends in a perpendicular or collinear way to its length. The pulses are detected by the deformation of the LPGs due to the passage of the pulses. An oscilloscope is used to detect the disturbance in the power transmitted by the LPGs. The speeds are calculated by measuring the distance between the LPGs and the difference in the time elapsed between the detections in both LPGs.

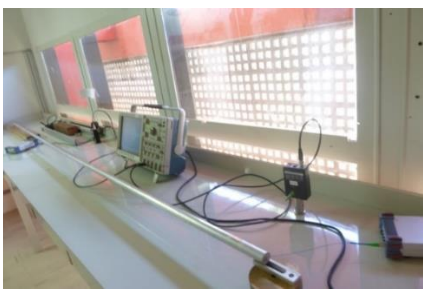

Fig. 1 Experimental arrangement used for the detection of transverse or longitudinal pulses traveling along an aluminum bar.

Figure 2 shows the optical signal recorded by the LPG sensors, separated by 2.55m. We observed that the average time of passage of a longitudinal pulse was 432 μs. The velocity of 5900 m/s was determined for these pulses.

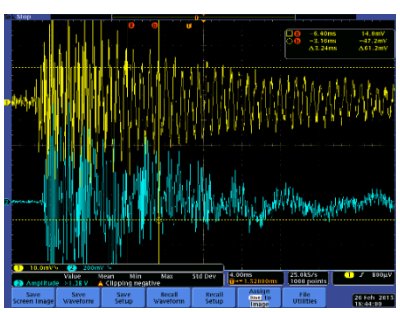

Fig. 2 Disturbance in the transmittance of the LPG gratings due to the passage of the traveling longitudinal pulse.

Figure 3 shows the optical signal recorded by the LPG sensors, separated by 2.55m. We observed that the average time of passage of a transverse pulse was 814.8 μs. The velocity of 3129.6 m / s was determined for these pulses.

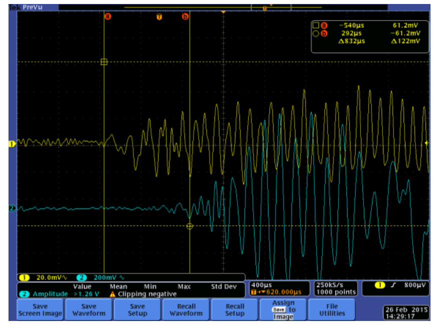

Fig. 3 Disturbance in the transmittance of the LPGs to the passage of a traveling transverse pulse.

Table 1 shows the record of times used by the pulses to travel the distance between the optical LPG sensors.

TABLE I

n	ΔT_L(μs)	ΔT_T (μs)
1	*432*	*830*
2	*448*	*830*
3	*448*	*800*
4	*378*	*830*
5	*420*	*832*
6	*440*	*808*
7	*444*	*808*
8	*444*	*810*
9	*424*	*810*
10	*444*	*790*
	$\Delta\overline{T_L}$ =432,2	$\Delta\overline{T_T}$ =814,8

L = 2,55m (distance between the centers of the LPGs)

V_L = 2,55/(432,2*10^{-6}) = 5900 m/s

V_T = 2,55/(814,8*10^{-6}) = 3129,6 m/s

A Young's modulus E = 68.967 × 10^9 Pascals was found for Aluminum using this technique. Also, values of the rigidity constant G = 26.432 × 10^9 Pascal and a Poisson module s = 0.3046. The values found for the elastic constants of the Aluminum using our technique are in agreement with the values of the Young's module, of rigidity and Poisson determined by other techniques [7-10].

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